

INFLATION MODEL BUILDING IN MODULI SPACE*

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A self-consistent modular cosmology scenario and its testability in view of future CMB experiments are discussed. Particular attention is drawn to the enhanced symmetric points in moduli space which play crucial roles in our scenario. The running and moreover the running of running for the cosmic perturbation spectrum are also analyzed.

1. Inflation model building

The important questions we would like to answer in inflation model building are

*What is the inflaton field?
What are its properties?*

To try to answer these questions, we need to consider what is a natural particle theory framework to discuss the dynamics of the early Universe.

(i) *What fields were there in the early Universe?*

The most promising candidate to describe the physics of the early universe is string theory, and string theory has many flat directions whose potentials vanish in exact supersymmetry. The fields parameterizing such flat directions are called moduli, and we shall discuss if a moduli field can realize a

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successful inflationary scenario or not.

(ii) *What does a modulus potential look like?*

The properties of moduli fields are heavily dependent on the way supersymmetry is broken. In the following, we discuss a hidden sector symmetry breaking scenario, in which the generic form of the moduli potential becomes

$$V(\phi) = M_s^4 F\left(\frac{\phi}{M_p}\right) \quad (1)$$

where M_s is the supersymmetry breaking scale and F is a dimensionless function with of order unity coefficients. In particular, for gravity mediated supersymmetry breaking, $M_s \approx 10^{10-11} \text{ GeV}$ and the mass of modulus ϕ consequently becomes $m_\phi = M_s^2/M_p \approx 10^{2-3} \text{ GeV}$. Note, as is usually the case for supergravity inflation, the slow-roll parameter V''/V is of order unity, which is a generic problem for supergravity inflation unless we choose a special form of Kähler potential.

2. Can a consistent inflationary scenario be realized in this natural context?

The detailed discussion of a possible self-consistent modular cosmology scenario based on this natural simple particle theory setup is given in [1]. We shall concentrate on the predicted cosmic perturbations for this scenario in this article. Note that we will consider a single modulus field which is complex. This does not mean we add an additional degree of freedom (namely angular component in addition to the radial component) because all scalar fields are complex in supersymmetry. The overview of the dynamics of the inflaton modulus is the following.

Near a maximum, the potential of Eq.(1) has the form^a

$$V(\phi) = V_0 - \frac{1}{2} m_\phi^2 |\Phi|^2 + \dots, \quad (2)$$

where $V_0 \sim M_s^4$ and $m_\phi \sim M_s^2$. Our scenario starts with an eternally inflating universe consisting of an ensemble of eternally inflating extrema throughout the field space of string theory, which avoids the Brustein-Steinhardt problem². Our local universe is a region where the field rolled down from its maximum to escape from the eternally inflating region, and

^aThis form relies on the maximum being a point of enhanced symmetry. See [3] for details.

the observable cosmic perturbations are produced while the field rolls down to its minimum. For a single complex modulus

$$\Phi = \frac{1}{\sqrt{2}} \phi e^{i\theta} , \quad (3)$$

the total curvature perturbations arise from radial and angular component fluctuations ⁴

$$\mathcal{R}_c = \delta N = \frac{\partial N}{\partial \phi} \delta \phi + \frac{\partial N}{\partial \theta} \delta \theta . \quad (4)$$

Before presenting the detailed results in Sec 2.2, simple estimates tell us that the contribution of radial component fluctuations to the final curvature perturbations is

$$\frac{\partial N}{\partial \phi} \delta \phi = -\frac{H}{\dot{\phi}} \delta \phi \sim \frac{H}{\phi} , \quad (5)$$

and that for the angular component is

$$\frac{\partial N}{\partial \theta} \delta \theta \sim \frac{\partial N}{\partial \theta} \frac{H}{\dot{\phi}} . \quad (6)$$

A crucial point here is that the angular component perturbations dominate for a large enough $\frac{\partial N}{\partial \theta}$.

2.1. *Points of enhanced symmetry*

The existence of points of enhanced symmetry is a robust and unique feature of moduli space. At those special points, some fields (matter, gauge, or moduli) become massless and become massive away from it, being Higgsed by the modulus that parameterizes the distance from the point of enhanced symmetry. At such a point of enhanced symmetry, the couplings of the modulus to those light degrees of freedom will cause the moduli mass to renormalize as a function of ϕ . This can turn the mass squared of the modulus positive for small ϕ , shifting the maximum of the potential out to some finite value $\phi = \phi_*$. The modulus maximum now becomes a rim and the field starts rolling down from there. The expression for the angular component fluctuation given in Eq.(6) had a steep spectrum because ϕ in the expression $H/\dot{\phi}$ changes rapidly due to non-slow-roll. It now, taking account of the loop corrections in the potential, goes like H/ϕ_0 where ϕ_0 is radius of the rim maximum of the potential, making the spectrum flat. The remaining problem is if large $\partial N/\partial \theta$ is probable, or, if the initial angle θ corresponding to large $\partial N/\partial \theta$ is probable. It is indeed probable, if we

consider that the regions corresponding to those initial angles leading to large $\partial N/\partial\theta$ would undergo greater expansion and hence occupy a larger volume at late times.

It turns out, as discussed in detail in [3], that the renormalized potential induced by the effects of light degrees of freedom at the saddle points identified as enhanced symmetric points will dynamically select the desirable initial angle for the inflaton modulus.

2.2. Observable predictions

In [3], we performed detailed analytic and numerical calculations for our modular cosmology scenario without assuming slow-roll conditions. The perturbation spectrum in our model has negligible deviations from scale invariance over a wide range of k with running becoming significant at (very) small (but possibly observable) scales. The form of the spectral index turned out to be a simple polynomial form

$$n - 1 = Ak^\alpha \quad (7)$$

Its running and running of running

$$\frac{dn}{d\ln k} = \alpha Ak^\alpha, \quad \frac{d^2n}{(d\ln k)^2} = \alpha^2 Ak^\alpha \quad (8)$$

illustrate our theoretical expectations that the usually assumed hierarchy $|n''| \ll |n'| \ll |n - 1| \ll 1$ is valid only for a limiting region of parameter space, $\alpha \ll 1$, where the running would in any case be small, while for a wider range of α the running is significant but so is the running of the running, $|n''| \sim |n'| \sim |n - 1|$.

It is also worth noting that Eq. (7) indicates that the running and the running of running are expected to be most significant toward smaller scales, i.e. negligible n' at large scales does not necessarily guarantee the absence of running in the spectrum, which cannot be taken into account if we ignore n'' . Thus it is crucial for our observations to probe the smallest possible scales to search for a signal of running.

3. Discussion

There are several works which try to explain the apparent discrepancy between the natural energy scale of the moduli potential ($\sim 10^{10-11}\text{GeV}$) and the energy scale of inflation ($\sim 10^{15}\text{GeV}$) obtained in some simple inflation models. They tend to try to find a non-trivial mechanism to scale up the

energy scale of the modulus potential to the GUT scale to match the amplitude of fluctuations with observations. We, however, instead stuck with the value of $\sim 10^{10-11}\text{GeV}$ and saw if our natural particle theory setup leads to the observationally consistent inflationary scenario. We discussed the cosmic perturbations in our scenario, which gives one of the most stringent constraints on an inflation model, with a particular emphasis on the possible running of running which in general cannot be ignored for the consistency of perturbation calculations.

In addition to the problems discussed in this article, another well-known and long-standing problem is the cosmological moduli problem. We proposed a baryogenesis scenario following thermal inflation in [5] to make this aspect of modular cosmology self-consistent too. Besides these phenomenological aspects of modular cosmology, more fundamental problems, such as moduli stabilization, are also under intense investigation⁶. The ubiquitous existence of moduli is a generic prediction of string/M-theory, and the realization of a successful modular cosmology scenario would be worth further study.

References

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